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Dynamic Models for Torque Converter Equipped Vehicles

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ABSTRACT

A dynamic model for the coupled engine-torque converter-vehicle system was developed to allow mathematical analysis and performance prediction for engine or powertrain controls in torque converter equipped vehicles.

The principal contribution of this work is the derivation and justification of a static non-linear terminal model of the torque converter. This model, when approximated, allows simple coupling of available engine, geartrain, and vehicle models.

The resulting differential equations provide a reasonably simple, physically justified model for engine and vehicle behavior when driven by large signal variations in either engine net torque output or vehicle load.

INTRODUCTION

Control system design usually includes the use of simplified models for the controlled physical process, to provide important information concerning system structure, stability, required control rate, and response. Recent work available in the automotive literature¹ has concentrated on the engine, with models which vary as a function of operating point used as the basis for linear control design.² Earlier vehicle modelling dealt with either dynamic description for use in automated control of roadways,³ or computer simulations of engine and vehicle without electronic control of the powerplant.⁴⁻⁶

Automotive control design is now turning from exclusive use of powerplant control to achieve desired emissions and fuel performance toward inclusion of the transmission. Initial work in engine control was directed toward achieving control over the engine's component subsystem (A/F ratio, spark timing, etc.). When control was achieved, emphasis shifted toward reaching more global performance objectives (optimization of fuel vs emissions using calibration refinements, etc.).⁷⁻⁸ Powertrain control is at the first stage.⁹ Shift control and shift pattern contouring will surely be followed by controls aimed at performance improvement for the entire powerplant/powertrain system.

To extend known control analysis and optimization methods in this direction, a link must be made between available engine and vehicle models. This is complicated for most passenger vehicles used in the United States because they incorporate a torque converter: a device whose operation and analytical description may not be familiar to the control engineer.

This work provides the link for this class of vehicles, providing non-linear terminal models for the torque converter. Using them, simple engine and vehicle models are coupled to provide an analytical model for combined engine-vehicle motion.

List of Symbols

Torque Converter

- | | |
|-----------------------------------|---|
| T_p, T_t, T_r | - torque converter pump, turbine and reactor torques. |
| $\omega_p, \omega_t, \omega_r$ | - converter pump, turbine, and reactor angular velocities. |
| TR | - converter torque ratio = T_t / T_p |
| K | - converter input capacity factor
= $\omega_p / \sqrt{T_p}$ |
| Q | - volume flow rate through the the converter. |
| ρ | - transmission fluid density. |
| r_{px}, r_{te} , etc. | - radius from converter axis to pump exit, turbine entrance, etc. First subscript indicates pump (p), turbine (t), or reactor (r). Second indicates element entrance (e) or exit (x). |
| α_{px}, α_{te} , etc. | - converter blade angle, SAE System A, subscript notation as before. |
| A_{px}, A_{te} , etc. | - converter cross-sectional area normal to converter annular flow Q . |
| L_{px}, L_{te} , etc. | - flow rate of rotational angular momentum. |

List of Symbols (contd)

P_t, P_p	- turbine, pump input powers.
$\Delta p_p, \Delta p_t$	- total pressures from turbine, pump input powers.
C_f	- friction coefficient for flow through the converter.
Δp_s	- shock loss of total pressure.
V_T	- fluid rotational or tangential velocity.
s	- torque converter speed ratio = ω_t / ω_p .

Engine-Vehicle Models

I_k	- equivalent inertia of vehicle and gearbox seen by converter turbine in gear k, including turbine inertia.
D_k	- equivalent vehicle damping in gear k.
A_k	- equivalent aerodynamic drag torque coefficient in gear k.
T_k	- rolling resistance torque referred to converter turbine in gear k.
I_e	- engine and converter pump inertia.
T_{net}	- engine net output torque, available to drive the vehicle or accelerate the engine and vehicle.
ω_e	- engine angular velocity, typically equivalent to converter pump speed ω_p .

A more complete description and derivation of the engine and vehicle model terminology is presented in the Appendix.

Motivation-Torque Converter Modelling

Control engineers normally deal with systems in which there are well defined explicit functional relationships between input and output variables. The preferred system model is one which directly relates the "across" variables at a system's ports (voltages, pressures, forces) to the "through" variables (currents, flows, motions), with internal feedback and port to port coupling within a modelled subsystem usually present. This is an underlying assumption behind block diagram and transfer function analysis used in control, and the familiar "Z", "Y", "h", and "s" parameters used in electronics.

Such block diagram or terminal equivalent models for each part of a system allow orderly interconnection and mathematical analysis of component interactions. With suitable restrictions

in the scope and detail of each model, this can be done for the engine, gearbox, and vehicle portions of an automobile. Addition of a terminal equivalent for the torque converter would allow interconnection to form a complete analytical engine/vehicle model, which could be elaborated or simplified, depending upon the application.

Unfortunately, a terminal equivalent is not the most compact form in which to describe the operation of torque converters or fluid couplings. The most typically used compact static representation is a joint plot of torque ratio and input capacity factor versus speed ratio.¹⁰ That is:

$$TR \triangleq T_t / T_p = f_1(\omega_t / \omega_p),$$

$$K \triangleq \omega_p / \sqrt{T_p} = f_2(\omega_t / \omega_p);$$

where TR = ratio of output to input torque as a function of the ratio of output to input speeds

K = input speed divided by input torque square root as a function of the ratio of output to input speeds.

T_p = pump (input) torque,

T_t = turbine (output) torque,

ω_p = pump (input) speed,

ω_t = turbine (output) speed.

This standard form is useful for comparing converters, and has been the basis for iterative numerical solutions used in computerized simulations.⁴⁻⁶ It does not, however, convey any insight to the control engineer about the nature of the internal coupling or feedback between converter input and output, or directly lend itself to control analysis.

The approach followed in this work is to start from simple physical models which capture the structure of the torque converter. These models are then recast into the form:

$$T_t = f_3(\omega_p, \omega_t), \quad T_p = f_4(\omega_p, \omega_t).$$

These are the desired explicit terminal relations between "across" variables (torques) and "through" variables (speeds). Since they will be derived from physical models, many alternative methods can be used to approximate them, with insight into the nature and applicability of the approximation.

Modelling The Torque Converter

A short discussion of the basics of fluid coupling and torque converter operation will now be presented to introduce hydrodynamic drives and illustrate analysis techniques for them. Enough detail is included to justify the terminal model

derivation and subsequent approximation. Additional descriptions of these analysis methods may be found in references [10-17].

Torque converters and fluid couplings are based upon the simple physical principle that the time rate of change of angular momentum of the fluid passing through a stationary or rotating element of a turbomachine must equal the torque applied to that element. Rotational angular momentum transferred to circulating fluid by the blades of a pump is carried by the fluid to a turbine, where it is absorbed by the turbine blades to produce output torque. This intuitive explanation¹¹ will now be expanded, beginning with the simplest hydrodynamic drive: the fluid coupling.

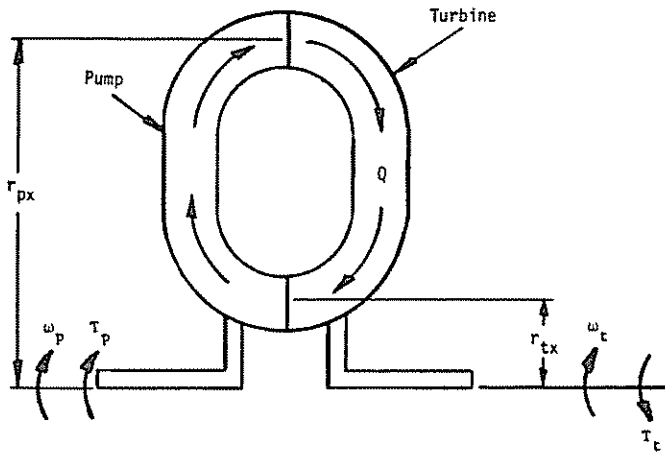


Figure 1 - Simplified Core Type Fluid Coupling - Half Cross Section.

FLUID COUPLING MODEL

An idealized fluid coupling with flat blades is shown in Fig. 1. Constant speed operation is assumed. Pump rotation due to applied torque T_p induces an annular volume flow rate Q through the pump and turbine elements, and a tangential velocity at the pump exit equal to the product of angular velocity ω_p and pump exit radius r_{px} . This tangential velocity times the pump exit radius is the tangential angular momentum per unit mass leaving the pump exit. Multiplication by fluid density and volume flow rate gives the exit angular momentum flow rate from the pump:

$$\dot{L}_{px} = \rho Q r_{px}^2 \omega_p$$

Performing a similar calculation at the turbine exit:

$$\dot{L}_{tx} = \rho Q r_{tx}^2 \omega_t$$

The rate at which angular momentum enters the turbine is the rate at which it leaves the pump. The difference gives the time rate of change of angular momentum across the turbine and the turbine torque.

$$T_t = \rho Q (\omega_t r_{tx}^2 - \omega_p r_{px}^2)$$

A similar argument gives the pump torque:

$$T_p = \rho Q (\omega_p r_{px}^2 - \omega_t r_{tx}^2)$$

The torques are of equal magnitude but opposite sign, descriptive of the power flow through the device. The pump input power is the product of pump torque and angular velocity:

$$P_p = \rho Q \omega_p (\omega_p r_{px}^2 - \omega_t r_{tx}^2)$$

which is positive, indicating power flow into the fluid coupling. The turbine input power is negative, indicating power flow from the fluid:

$$P_t = \rho Q \omega_t (\omega_t r_{tx}^2 - \omega_p r_{px}^2)$$

Although angular momentum has been conserved, mechanical power was not. Summing pump and turbine input powers gives the rate at which the coupling dissipates energy under steady state conditions.

$$P_p + P_t = \rho Q (\omega_p^2 r_{px}^2 - \omega_p \omega_t r_{tx}^2 + \omega_t^2 r_{tx}^2 - \omega_t \omega_p r_{px}^2)$$

For non-zero flow, this dissipation is only zero when pump and turbine speeds are equal.

Although we have a variety of relationships between the torques, speeds, and flow rate for the coupling, we cannot explicitly solve for the magnitude of the torques, because we have not solved for the flow rate in terms of the angular speeds.

To find the flow rate, we use the fact that for steady flow conditions, the energy added to a fluid particle in a complete circuit around the coupling must be zero. Flow energy and losses are commonly described in terms of total pressure.^{18,19} Dividing pump and turbine input powers by the volume flow rate gives the net total pressure added from external inputs:

$$\frac{P_p}{Q} + \frac{P_t}{Q} \triangleq \Delta p_p + \Delta p_t = \rho (\omega_p^2 r_{px}^2 - \omega_p \omega_t r_{tx}^2 + \omega_t^2 r_{tx}^2 - \omega_t \omega_p r_{px}^2)$$

Total pressure losses around the flow circuit are ascribed to two sources. The first is fluid friction, taken as proportional to the square of the volume flow rate: $\Delta p_f = 1/2 \rho C_f Q^2$.

The second source, given the name "shock loss"¹⁹ is due to the sudden change in velocity as fluid crosses the boundary between one blade system and the next. Irreversibility in this process gives rise to energy losses, analogous to the total pressure loss in sudden expansions or contractions of cross section in pipe flow. This loss is proportional to the square of the difference in tangential velocities and, hence, to the square of the difference in rotational velocities.

Summing the shock losses due to the two transitions the fluid makes in a circuit around the coupling:

$$\Delta p_s = 1/2 \rho (r_{px}^2 + r_{tx}^2) (\omega_p - \omega_t)^2$$

All this energy will be assumed lost, although careful design of blade profiles can lower this loss.

Adding the pressure inputs, subtracting the pressure losses, requiring that the total pressure added be zero for steady operation leads to the following expression:

$$1/2 \rho (r_{px}^2 - r_{tx}^2) (\omega_p^2 - \omega_t^2) = 1/2 \rho C_f Q^2$$

This "compatibility"^{13,14} condition can be solved for the volume flow rate:

$$Q = \left(\frac{r_{px}^2 - r_{tx}^2}{C_f} \right)^{1/2} (\omega_p^2 - \omega_t^2)^{1/2}$$

Substitution into the expression for torque yields:

$$T_p = T_t = \rho \left(\frac{r_{px}^2 - r_{tx}^2}{C_f} \right)^{1/2} (\omega_p^2 - \omega_t^2)^{1/2} (r_{px}^2 \omega_p - r_{tx}^2 \omega_t)$$

This is a terminal equation for torques in terms of angular velocities which illustrates the methods used to solve for the terminal behavior of the torque converter. It can be generalized to account for the effect of blade angles, present in most actual couplings and torque converters.

If blade entrance or exit angles differ from zero, the expressions for tangential velocity directly involve the flow rate, because the blade can convert fluid flow rate into additional tangential velocity in the plane of rotation of the pump or turbine.

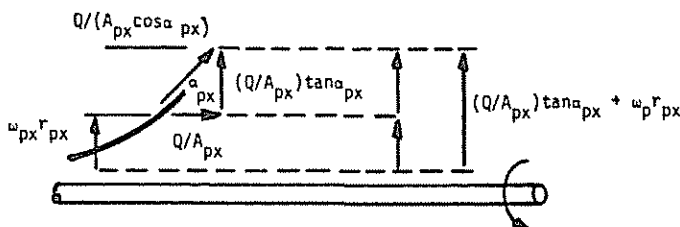


Figure 2 - Effect of Non-Zero Blade Angle on Tangential Exit Velocity.

Figure 2 is drawn for fluid leaving the coupling's pump, with volume flow rate Q normal to the pump exit area A_{px} . The normal component of flow velocity is Q/A_{px} , while the relative tangential component of velocity is $(Q/A_{px}) \tan \alpha_{px}$ if the fluid is assumed to exit tangent to the blade. Adding blade exit absolute velocity gives the fluid absolute tangential velocity:

$$V_{Tpx} = \frac{Q}{A_{px}} \tan \alpha_{px} + \omega_p r_{px}$$

Performing operations analogous to those done for flat blades, the pump exit angular momentum flow has been changed by the blade angle.

$$\dot{L}_{px} = \rho Q \left(\frac{Q}{A_{px}} \tan \alpha_{px} + \omega_p r_{px} \right) r_{px}$$

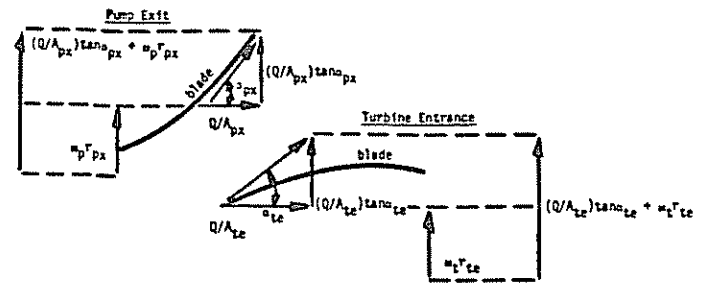


Figure 3 - Change in Tangential Velocity Across Pump-Turbine Boundary.

"Shock" losses of energy are also modified by non-zero blade angles. The velocity diagrams in Figure 3 show the interface between the pump exit and turbine entrance. The difference in tangential velocities is now:

$$\Delta V_T = Q \left(\frac{\tan \alpha_{px}}{A_{px}} - \frac{\tan \alpha_{te}}{A_{te}} \right) + \omega_p r_{px} - \omega_t r_{te}$$

The shock loss in total pressure becomes:

$$\Delta p_{s_{p-t}} = 1/2 \rho \left[Q \left(\frac{\tan \alpha_{px}}{A_{px}} - \frac{\tan \alpha_{te}}{A_{te}} \right) + \omega_p r_{px} - \omega_t r_{te} \right]^2$$

These blade angle effects complicate the analysis by addition of terms in flow rate squared to the torque equations, and the product of flow and angular velocity to the compatibility equation. They do not, however, alter the method used to find the terminal equivalent of the torque converter: by direct solution for the volume flow rate.

Torque Converter Model

Torque converters differ from the fluid coupling by the addition of a third stationary set of blades, the reactor or stator, to the flow circuit. This is shown in Fig. 4. Fluid leaving the turbine element is deflected by the reactor so that, at the reactor exit, fluid tangential velocity is in the direction of pump rotation. Since the pump exit angular momentum flow is the sum of input from the reactor and that added by the pump, the turbine sees a larger angular momentum flow than that due to the pump alone. This effectively multiplies the pump input torque.

The additional torque is supplied by the reactor, which is fixed to the converter axis. Since the reactor does not rotate, it does no work. The reactor is typically mounted on a one way clutch or sprag clutch, so that it can only add, not subtract torque. When the entrance and exit momentum flows of the reactor are equal, it freewheels. At this point, the converter behaves as a fluid coupling.

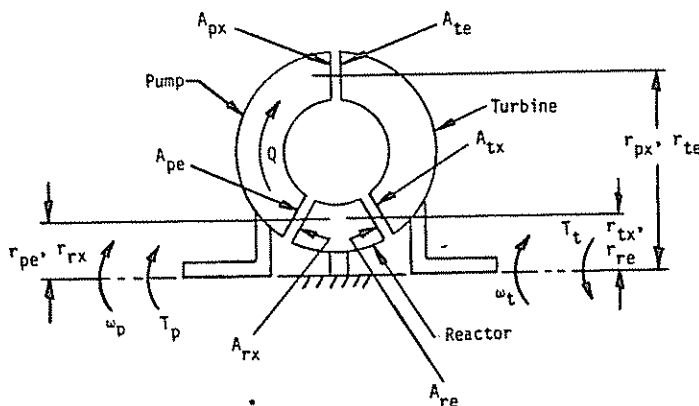


Figure 4 - Three Element Torque Converter Cross Section.

In the detailed exposition which follows, subscript notation is as before, with the first subscript indicating the element (p = pump, t = turbine, r = reactor) and the second indicating the element entrance (e) or exit (x). Entrance and exit are defined by the normal direction of annular flow with the converter delivering power to the turbine. SAE Blade System A²⁰ is used to describe blade angles, with positive angles indicating deflection of flow in the direction of positive angular velocity. Translating an analysis due to Eksergian¹⁴ into this system, the pump and turbine torque equations can be shown to be:

$$T_p = A\omega_p Q + BQ^2,$$

$$T_t = A\omega_t Q - C\omega_t Q + DQ^2$$

where to simplify the equations, the following substitutions have been made.

$$A = \rho r_{px}^2,$$

$$B = \rho \left(\frac{r_{px} \tan \alpha_{px}}{A_{px}} - \frac{r_{rx} \tan \alpha_{rx}}{A_{rx}} \right)$$

$$C = \rho r_{tx}^2$$

$$D = \rho \left(\frac{r_{px} \tan \alpha_{px}}{A_{px}} - \frac{r_{tx} \tan \alpha_{tx}}{A_{tx}} \right)$$

The compatibility equation to be solved for volume flow rate is:

$$IQ^2 + (H\omega_t - G\omega_p)Q - (E\omega_p^2 + F\omega_t^2) = 0$$

subject to the following substitutions which allow this simplification.

$$E = 1/2\rho(r_{px}^2 - r_{pe}^2)$$

$$F = 1/2\rho(r_{tx}^2 - r_{te}^2)$$

$$G = \rho \left(\frac{r_{px} \tan \alpha_{te}}{A_{te}} - \frac{r_{pe} \tan \alpha_{pe}}{A_{pe}} \right)$$

$$H = \rho \left(\frac{r_{te} \tan \alpha_{te}}{A_{te}} - \frac{r_{tx} \tan \alpha_{re}}{A_{re}} \right),$$

$$I = 1/2\rho \left[\frac{(\tan \alpha_{px} - \tan \alpha_{te})^2}{A_{px}^2} \right.$$

$$+ \frac{(\tan \alpha_{tx} - \tan \alpha_{re})^2}{A_{tx}^2}$$

$$+ \left. \frac{(\tan \alpha_{rx} - \tan \alpha_{pe})^2}{A_{rx}^2} + C_f \right]$$

Solving for the volume flow rate,

$$Q = \frac{-(H\omega_t - G\omega_p)}{2I} + \frac{\sqrt{(H\omega_t - G\omega_p)^2 + 4I(E\omega_p^2 + F\omega_t^2)}}{2I}$$

The positive sign for the square root is chosen to correspond to the assumed positive direction of flow used to define angular momentum flow, torques, and power flow through the converter. Reversal of flow, as in overrun, would require interchange of the angles, areas, and radii defining the coefficients.

Substitution of the volume flow rate solution into the equations for pump and turbine torques gives exact, explicit terminal relations for them in terms of angular speeds.

$$T_p = \left[A\omega_p + \frac{B(G\omega_p - H\omega_t)}{I} \right] \cdot \left[\frac{\sqrt{(H\omega_t - G\omega_p)^2 + 4I(E\omega_p^2 + F\omega_t^2)}}{2I} - \frac{(H\omega_t - G\omega_p)}{2I} \right] + \frac{B}{I}(E\omega_p^2 + F\omega_t^2)$$

$$T_t = \left[A\omega_p - C\omega_t + \frac{D(G\omega_p - H\omega_t)}{I} \right] \cdot \left[\frac{\sqrt{(H\omega_t - G\omega_p)^2 + 4I(E\omega_p^2 + F\omega_t^2)}}{2I} - \frac{(H\omega_t - G\omega_p)}{2I} \right] + \frac{D}{I}(E\omega_p^2 + F\omega_t^2)$$

These expressions are exact, within the scope of the simplified one dimensional mean flow path analysis used as the starting point. Substitution of numerical values for the parameters defining each coefficient would allow prediction of the static performance of a given torque converter, in an approximate sense. Although the equations are too complex to be used directly for control analysis, they do illustrate a simple fact: the torque converter is not a continuously variable transmission, in which the input speed is adjustable for a given output power level and speed.

If a required output torque and speed are specified, the turbine torque equation can be solved for the required value of input speed. Use of the values for pump and turbine speed allows prediction of the required value for input torque. In two-port terms, the "load impedance" uniquely determines the "input impedance."

Fluid Coupling Operation of the Torque Converter

To improve torque converter efficiency, the reactor element is allowed to free wheel when its torque reaction on the fluid is zero. At this point, the torque converter behaves as a fluid coupling, with an additional loss due to the extra transition between the turbine exit and pump entrance.

An equation written for the change of angular momentum across the reactor (which must be zero, because the reactor can deliver no torque when free-wheeling) can be used to solve for reactor angular velocity. This allows formation of equations for pump and turbine torques, and an approximate compatibility condition on total pressure.

It can be shown that the previously derived equations for turbine torque and flow rate adequately describe the torques and flow rate through the converter in the coupling mode. Pump and turbine torque are equal in a fluid coupling.

The only difference is a change to the coefficients H and I: in the coupling mode,

$$H' = \rho \left(\frac{r_{px} \tan \alpha_{te}}{A_{px}} - \frac{r_{tx} \tan \alpha_{pe}}{A_{rx}} \right),$$

$$I' = 1/2\rho \left[\frac{(\tan \alpha_{px} - \tan \alpha_{te})^2}{A_{px}^2} + \left(\frac{\tan \alpha_{rx}}{A_{rx}} - \frac{\tan \alpha_{re}}{A_{tx}} \right)^2 + \left(\frac{\tan \alpha_{pe}}{A_{rx}} - \frac{\tan \alpha_{tx}}{A_{tx}} \right)^2 + C_f \right].$$

A third case - overrun - will not be presented. Analogous methods can be used to produce exact expressions for this mode of converter operation. Because the reactor blade is free wheeling, they could be expected to be of the same form as those for the fluid coupling mode, without torque multiplication in the reverse direction of power flow.

Approximate Static Terminal Models for the Torque Converter

The preceding derivation provides exact terminal equivalents for the torque converter. The expressions for pump and turbine torque which were its outcome are too complex, however, to be used directly for control design purposes. Approximations will now be given which still capture the structure of the converter, but greatly simplify its analytical description.

To simplify the expressions for pump and turbine torque, an approximation for the volume flow rate was sought. Rewriting the expression for the volume flow rate through the converter,

$$Q = \frac{(G\omega_p - H\omega_t)}{2I} + \frac{\sqrt{(H\omega_t - G\omega_p)^2 + 4I(E\omega_p^2 + F\omega_t^2)}}{2I}.$$

This can be approximated by performing a one dimensional Taylor expansion in the variable ω_t about the point $\omega_t^* = G\omega_p/H$. If the turbine speed ω_t equals $\omega_t^* = G\omega_p/H$, the flow rate varies linearly with pump speed:

$$Q(\omega_p, \omega_t^*) = \omega_p \sqrt{\frac{E + FG^2/H^2}{I}}$$

This linear dependence is not surprising. Flow rate in pipes is commonly modelled as proportional to the square root of pressure difference:

$$Q \propto \sqrt{\Delta p}.$$

In the torque converter, when $\omega_t = \omega_t^*$, $\Delta p = E\omega_p^2 + F\omega_t^2$, which is the difference in pressure imposed by rotation of the pump and turbine. Using the constraint $\omega_t^* = G\omega_p/H$ gives:

$$\Delta p = \omega_p^2 (E + FG^2/H^2),$$

so that Q is proportional to $\sqrt{\Delta p}$ as for flow in pipes.

The conclusion can be generalized. If there is a linear constraint connecting turbine and pump speed,

$$\omega_t = s\omega_p,$$

then the flow rate is linear with pump speed. The variable s is commonly given the name "speed ratio." Rephrasing, volume flow rate is linear with pump speed for constant speed ratio.

The choice of the constraint $\omega_t^* = G\omega_p/H$ is really arbitrary, done only to simplify the evaluation of derivatives in the Taylor series expansion. Performing the expansion about $\omega_t = \omega_t^* = G\omega_p/H$ gives:

$$\begin{aligned} Q(\omega_p, \omega_t) &= Q(\omega_p, \omega_t^*) + \frac{\partial Q}{\partial \omega_t} (\omega_t - \omega_t^*) \\ &+ 1/2 \frac{\partial^2 Q}{\partial \omega_t^2} (\omega_t - \omega_t^*)^2 + \dots \\ Q(\omega_p, \omega_t) &= \sqrt{\frac{E + FG^2/H^2}{I}} \omega_p \\ &+ \left(\frac{-H}{2I} + \frac{2FG}{\sqrt{4I(EH^2 + FG^2)}} \right) (\omega_t - G\omega_p/H) \\ &+ O(\omega_t - \omega_t^*)^2. \end{aligned}$$

Approximate expressions for flow rate are formed by truncation at either the first term (zero order model), or the second term (first order model). The expansion was stopped with the first order term because regression of actual converter data did not justify higher order terms, and the purpose of the work was to provide simple models. A zero order model would ignore the unloading, feedback effect of increasing turbine speed on flow rate and pump torque, and dynamic models derived from it would have the wrong structure.

Keeping first order terms,

$$\begin{aligned} \tilde{Q}_1 &= \left(\frac{E}{\sqrt{I(E + FG^2/H^2)}} + \frac{G}{2I} \right) \omega_p \\ &+ \left(\frac{FG}{H\sqrt{I(E + FG^2/H^2)}} - \frac{H}{2I} \right) \omega_t; \end{aligned}$$

which when simplified, takes the following form:

$$\tilde{Q}_1(\omega_p, \omega_t) = \alpha\omega_p + \beta\omega_t.$$

Using this in the expressions for torque makes the terminal equivalents take on quadratic forms in the vector of speeds $(\omega_p, \omega_t)^T$.

$$T_p = a_0\omega_p^2 + a_1\omega_p\omega_t + a_2\omega_t^2$$

$$T_t = b_0\omega_p^2 + b_1\omega_p\omega_t + b_2\omega_t^2.$$

These expressions with suitable coefficients provide a simplified analytical model for the torque converter in the converter mode, multiplying torque. Because the approximated equation does not change in either coupling or overrun modes, suitable adjustment of parameters would allow use of these forms for description of the converter in these modes.

In addition, the following simple representation for torque ratio follows from the torque equations and use of the first order flow model:

$$\frac{T_t}{T_p} = \frac{(A + D\alpha)\omega_p + (D\beta - C)\omega_t}{(A + B\alpha)\omega_p + B\beta\omega_t}$$

This is the ratio of two linear equations.

The error involved in these approximations will not be formally analyzed, but can be inferred from the regression of real converter data, which follows.

Regression Fit to Actual Converter Data

Verifying the connection between areas, angles, radii, etc., for a particular converter and its terminal properties was felt to be beyond the scope of this work. The approximate model structure was used as the basis for regression fits of actual converter data. Instead of deriving coefficients for the structure from converter physical data, regression of torque data using the structure was done to find coefficients for a given converter. This was more in keeping with the control modelling purpose of the work.

Data from a variety of converters was fitted using the model just derived. Regression using a linear combination of pump speed squared, turbine speed squared, and the cross product of pump and turbine speeds yielded excellent results for prediction of pump and turbine torques. Torque ratio was also well described by the ratio of two linear equations.

Regression was performed for turbine and pump torques using the following normalization by pump speed squared:

$$\frac{T_t}{\omega_p^2} = a_0 + a_1 s + a_2 s^2$$

The software used performed the linear regression:

$$(T - a_0) = \sum_{i=1}^3 a_i x_i$$

where $(x_1, x_2, x_3) = (\omega_p^2, \omega_p \omega_T, \omega_T^2)$.

In the un-normalized form, the algorithm could choose a_0 to lower the error between predicted and actual data. This produced formulas giving non-zero pump or turbine torques at zero angular speeds. There may be static or additional mechanical friction present in measured data, but it was not separately accounted for. This was judged to be a conservative approach toward evaluating the model. Regressions were performed for both converter and coupling modes of operation, with four different values of pump torque used in standard torque ratio versus speed ratio tests, together with a stall test at zero turbine speed. The lowest value of correlation coefficient for the data shown was .985, while the error between predicted and measured torques was typically within 3%. Figures 5, 6, and 7 show the results of the normalized regressions for pump torque, turbine torque, and torque ratio.

The variation of transmission fluid density and viscosity with temperature is well known.¹⁶ All data shown here was for converter oil inlet temperatures of 88°C (190°F), representative of its temperature in a warmed up vehicle. Variations of converter characteristics during warmup can be expected, but their analysis is beyond the scope of this work.

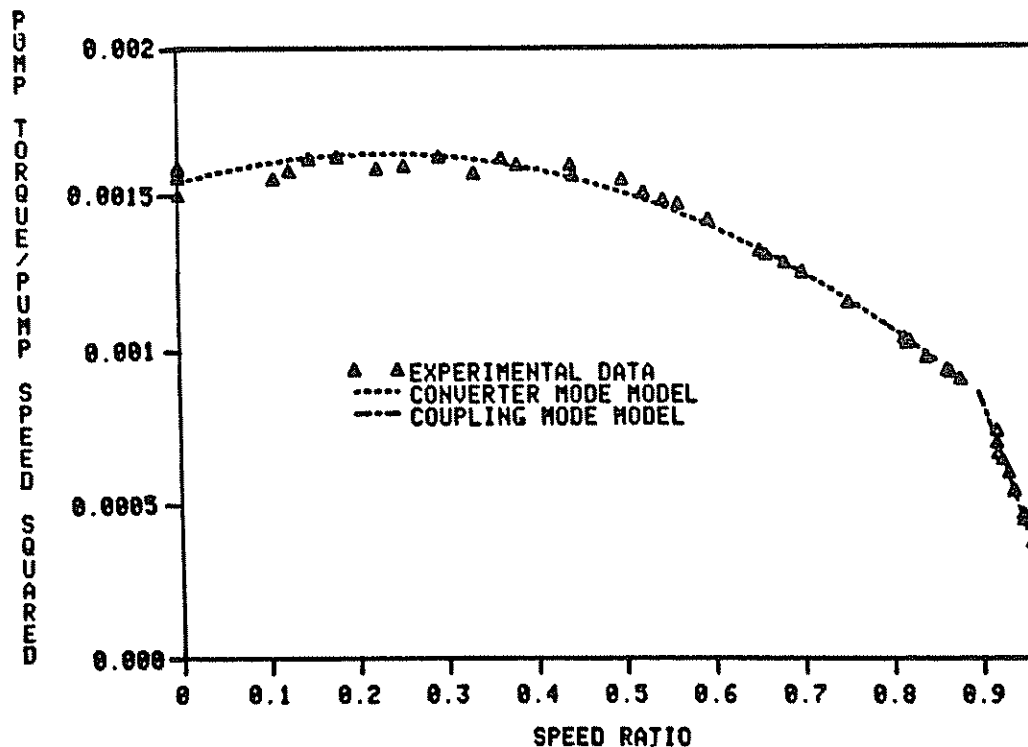


Figure 5 - Pump Torque Divided by Pump Speed Squared vs Speed Ratio. Comparison of Experimental Data With Simplified Models.

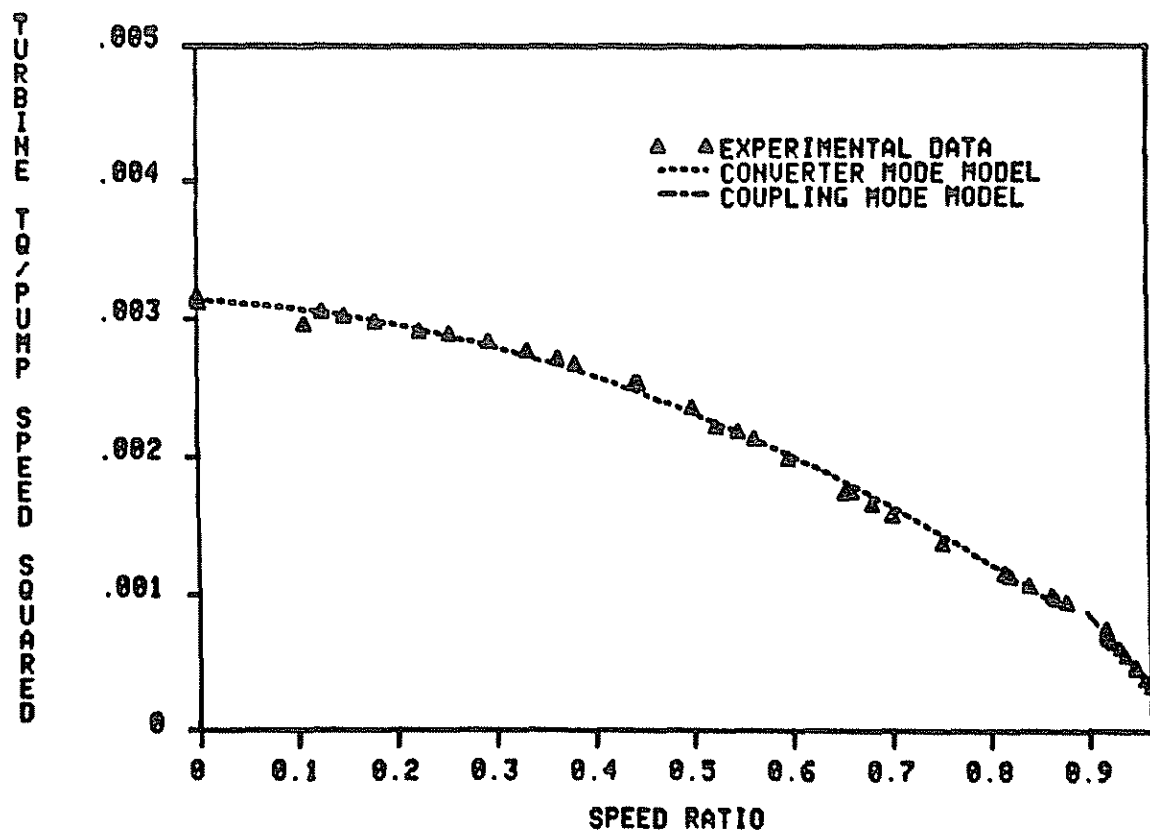


Figure 6 - Turbine Torque Divided by Pump Speed Squared vs Speed Ratio. Comparison of Experimental Data With Simplified Models.

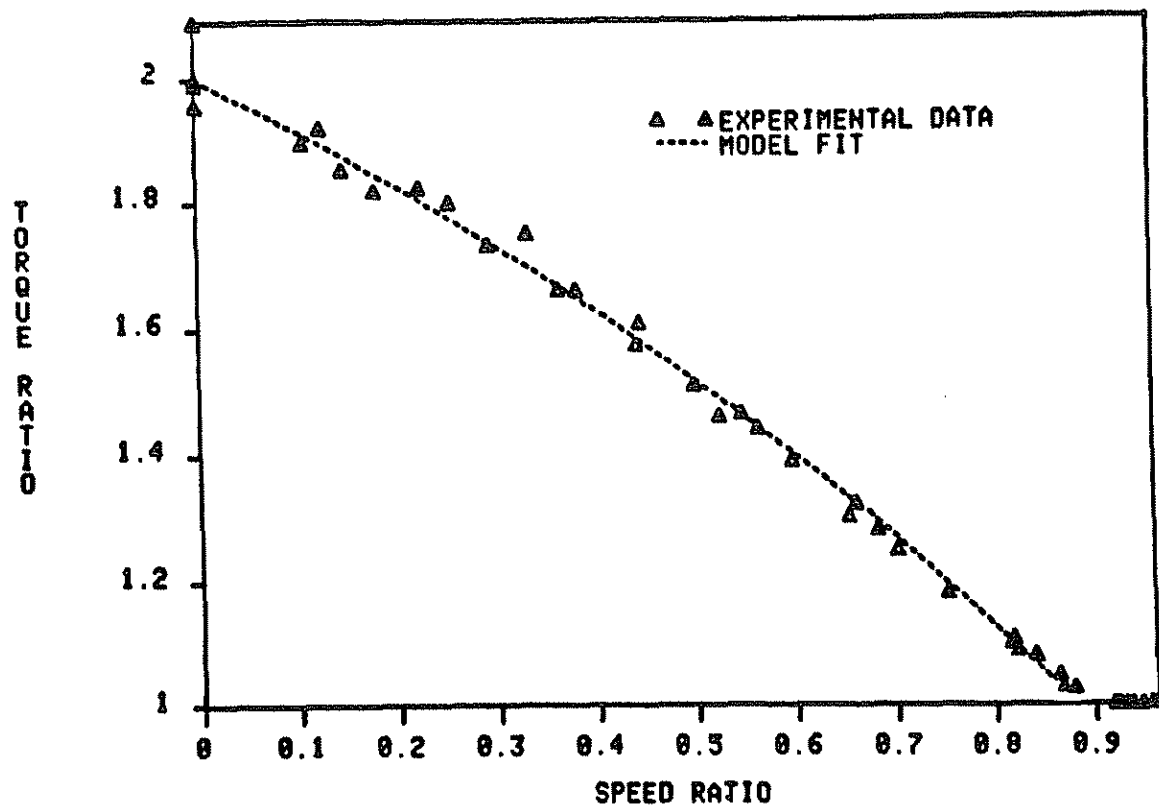


Figure 7 - Torque Ratio vs Speed Ratio. Comparison of Experimental Data With Simplified Model.

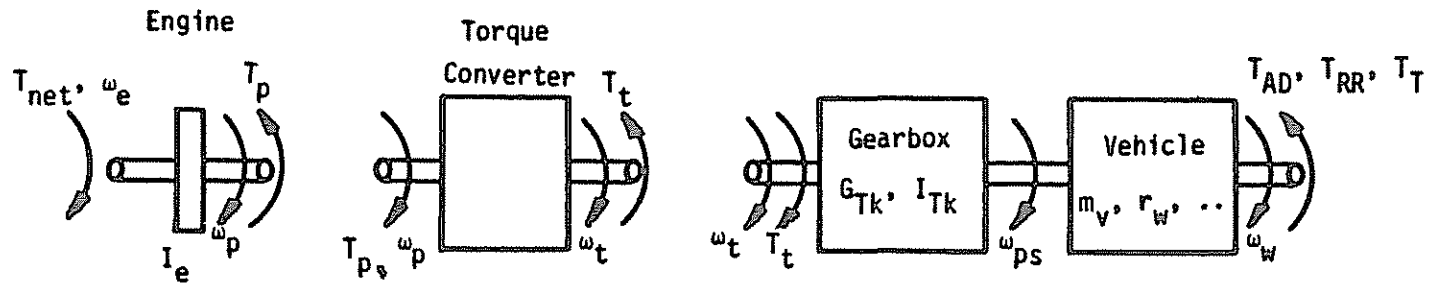


Figure 8 - Coupling the Engine and Vehicle Models Through the Converter.

Coupling the Models

To build a coupled engine-vehicle model, what remains is a straightforward substitution of the pump torque expression into a differential equation for engine motion, and the turbine torque into a turbine-gearbox-vehicle differential equation. These equations are included in the Appendix of this paper. Because the converter has been transformed into a terminal equivalent, the coupling is performed by a simple cascading of engine, converter, and gearbox-vehicle models; as illustrated in Fig. 8. In the interest of simplicity, the approximate representations for pump and turbine torque will be used.

Below the converter coupling point, the engine and vehicle obey the following differential equations.

$$\begin{aligned}
 I_k \dot{\omega}_t + D_k \omega_t + (A_k - b_2) \omega_t^2 - b_1 \omega_e \omega_t \\
 - b_o \omega_e^2 &= -T_k \quad (\text{turbine-vehicle}) \\
 I_e \dot{\omega}_e + a_o \omega_e^2 + a_1 \omega_e \omega_t \\
 + a_2 \omega_t^2 &= T_{\text{net}} \quad (\text{engine-pump})
 \end{aligned}$$

Above the coupling point, with the converter no longer multiplying engine torque, only a single set of coefficients is required to describe converter behavior (i.e., $a_1 = b_1 = a_1'$, etc.). This is reflected in the differential equations describing the system.

$$\begin{aligned}
 I_k \dot{\omega}_t + D_k \omega_t + (A_k - a_2') \omega_t^2 \\
 - a_1' \omega_e \omega_t - a_o' \omega_e^2 &= -T_k \quad (\text{turbine-vehicle}) \\
 I_e \dot{\omega}_e + a_o' \omega_e^2 + a_1' \omega_e \omega_t \\
 + a_2' \omega_t^2 &= T_{\text{net}} \quad (\text{engine-pump})
 \end{aligned}$$

This was the result sought: a non-linear dynamic model for engine speed and turbine (vehicle) speed driven by net engine torque and external forces.

DISCUSSION

Overrun and Sprags

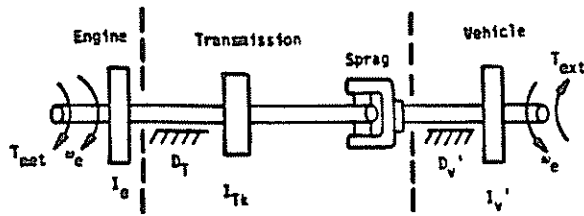
The relationships derived can be extended to provide a complete description of vehicle operation with the addition of a terminal model for the torque converter operating in the overrun mode. In this regime, turbine or output element speed exceeds that of the engine and pump, so reversal of flow within the converter could be expected. But the form of the analysis used and the approximations derived should be unchanged, allowing regression of overrun data to provide approximate models of the same type.

Care must be taken, however, to prevent misapplication of the models. In typical automatic transmissions, there is not always a direct mechanical connection between the output shaft and the turbine.²¹ With the transmission selector in "Drive", sprag clutches are used to supply unidirectional reaction torques to the internal gearsets of the transmission in 1st or 2nd gear. Should the driver remove his input to the engine with the transmission in this mode, the sprags unlock or overrun, preventing the reversal of torque in the propeller shaft. Thus, in "Drive", with 1st or 2nd gear engaged, the engine cannot be used to extract energy from the vehicle. In "Drive", with high gear engaged, a direct connection is typically present which provides engine braking.

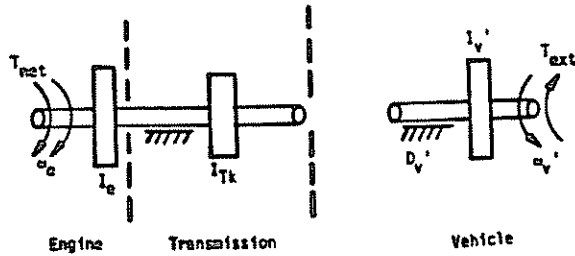
Should higher rates of energy dissipation or engine braking be desired, the "Low", "L₁", or "L₂" modes of the transmission can be used. While limiting the potential range of gears and raising the available rate of energy dissipation, these modes engage band clutches which can transmit bidirectional torques between the output shaft of the transmission and the converter turbine.

The models previously derived assumed the direct connection. With the addition of a third overrun model for the torque converter, they can provide an adequate description of vehicle behavior for control in Drive (top gear), selectable Low gears, and Reverse.

In Drive, with 1st or 2nd gear engaged, an obvious extension of the model can be made. To illustrate this, Fig. 9 is a simplified linear analog of the engine and vehicle system.



(a) Locked Sprag



(b) Free-Wheeling Sprag

Figure 9 - Simplified Linear Analog to Illustrate Sprag Operation.

Figure 9a shows the normal configuration, with the sprag or one-way clutch transmitting torque to drive the vehicle. Should engine net torque drop rapidly, the vehicle will decelerate. If the required deceleration is too large, requiring the transmission of negative torque, the sprag free-wheels, splitting the system into two component pieces (Fig. 9b).

The first is the engine, torque converter, and gearbox, driven by engine torque. The second is the vehicle, driven by external torques opposing its motion.

The condition defining this point can be easily derived. For the sprag to be locked, its transmitted torque must be positive. Therefore;

$$T_{\text{net}} - I_e \dot{\omega}_e - I_{Tk} \dot{\omega}_e - D_T \omega_e \geq 0$$

and

$$D_v \omega_e + I_v \dot{\omega}_e + T_{\text{ext}} \geq 0.$$

Using the second inequality;

$$\dot{\omega}_e \geq -(T_{\text{ext}} + D_v \omega_e) / I_v$$

for the locked state. The sprag will free-wheel if the deceleration is greater than that which could be induced by external torques and driveline drag alone. Using equality of the second condition in the inequality for engine net torque;

$$T_{\text{net}} \geq - \frac{(I_e + I_{Tk}) T'_{\text{ext}}}{I_v} + \omega_e (D_T - \frac{(I_e + I_{Tk})}{I_v} D_v).$$

This places a lower bound on engine net torque above which the sprag will be locked, and the models already derived are directly usable.

Reflecting these simple approximations into the non-linear models provides conditions on net torque for sprag locking and unlocking in Drive, 1st or 2nd gears. Below this lower bound, the sprag would free-wheel, splitting the system into two parts: (a) the engine, converter, and transmission mechanical components, and (b) the vehicle.

Quasi-Static Flow Assumption

The largest limiting assumption of the model in its present form is that the converter instantly achieves the steady state flow rate required by engine and turbine speeds. This is not true. It is also a limitation of most quasi-static engine simulation programs.⁵ Additional energy must be diverted from the engine into fluid kinetic energy within the converter during transients. The fluid must be described by a third differential equation.

This additional required kinetic energy reflects itself as small errors in the prediction of engine and turbine torques. Previous work on vibration damping^{14,17} showed these errors were negligible if the frequency of disturbances in velocity was held to less than twice per revolution. At idle, this is about 20 Hz, well above our present ability to affect average torque with electronic controls.

A sudden change in the boundary conditions on the converter (such as a ratio change in the gearbox) can induce large errors in the prediction of terminal torques. These effects are, however, of short duration.²² Similar effects can also be felt after sudden application of throttle after long part throttle coasts in Drive, high gear. A distinct delay in vehicle acceleration occurs because fluid motion from pump to turbine has either stopped or reversed directions.

These effects do not seriously limit the use of the current models for engine control analysis provided that suitable boundary conditions on engine and turbine speeds are imposed when shifting, or time intervals are prescribed for the fluid to change direction when crossing from powered to coasting operation.

Linearization

The differential equations for engine and vehicle presented here have been linearized to predict equivalent transfer functions for the vehicle system near an operating point. The utility of these methods, however, is tempered by the speed

with which the operating point can change using only a fraction of the power available from the engine. It is a rare individual who drives his car with small delta changes in engine torque. The method of linearization is, nevertheless, clearly useful for stability or driveability analyses.

Future Work

Additional work should include flow rate dynamics in the model. This adds additional terms to the equations for turbine and engine acceleration, and a third differential equation for volume flow rate. For most engine control work, they can be neglected. For shift control and control of converter clutch application, they cannot be.

Additional insight into control response requirements, rates, operating point stability, and sensitivity to calibration can be gained by using available engine models in conjunction with the work contained here.

CONCLUSIONS

Simple, physically-based quadratic form models can be used to provide terminal equivalents for the torque converter.

Regression of real converter data shows these models to be accurate, with index of determination greater than 0.985.

These converter models can be used to formulate coupled engine-converter-gearbox-vehicle models useful for control system analysis and design.

ACKNOWLEDGEMENTS

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APPENDIX

Vehicle Model

The vehicle is modelled³ as a mass m_v (kg) situated a distance above the roadway equal to the tire rolling radius r_w (m). This mass includes that of the tires. Suspension is assumed rigid, to prevent introduction of suspension dynamics into the present problem.

The vehicle moves above the roadway with a velocity v_v , and is subjected to several external forces which modify its motion: aerodynamic drag F_{AD} , tire rolling resistance F_{RR} , and force due to variations in the slope or surface of the terrain over which the vehicle travels F_T . These forces are opposed by a driving force F_w created by torque T_w .

Energy is also dissipated in overcoming viscous friction internal to the vehicle chassis. Static chassis and driveline friction must be overcome to start the vehicle moving, but will be assumed to be zero once the vehicle is underway. These forces do, however, fix the boundary conditions for engine speed and torque required to set the vehicle in motion.

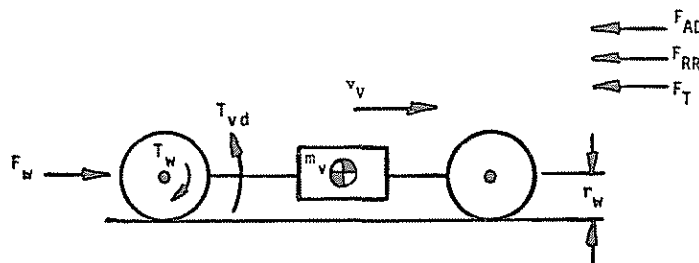


Figure A1 - Idealized Vehicle Model.

Aerodynamic drag is commonly modelled as varying in a quadratic manner with vehicle speed:^{13,23}

$$F_{AD} = 1/2 \rho_{Air} C_D A_F v_v^2$$

where

C_D = vehicle drag coefficient

A_F = vehicle frontal area (m²)

ρ_{air} = air density (kg/m³)

v_v = vehicle velocity (m/s).

A simple, low speed model for tire rolling resistance is that of a constant retarding force proportional to the load on the tire:²³

$$F_{RR} = m_v g C_{RR_{tires}}$$

where

m_v = vehicle mass (kg)

g = gravitational constant
(= 9.806 N/kg)

$C_{RR_{tires}}$ = dimensionless ratio of retarding force to tire load.

Typical values for $C_{RR_{tires}}$ are in the range of .01 to .02,²³⁻²⁵ with passenger car tires near the higher end of the range. As vehicle speed increases, tire radius increases and standing waves in the tire preclude use of this simple model.^{3,26}

Viscous drag torques, whatever their source or path through the chassis of the vehicle, will be assumed proportional to wheel rotational speed:

$$T_{VD} = D_v \omega_w$$

where

D_v = viscous drag torque coefficient (N·m·s)

ω_w = wheel angular velocity (s⁻¹)

The kinetic energy stored in the vehicle's motion is composed of two parts: translational energy and rotational energy stored in the wheels and other rotating parts of the driveline:

$$E_V = 1/2 m_V v_V^2 + 1/2 I_T \omega_w^2,$$

where

I_T = equivalent moment of inertia of tires and driveline ($\text{kg}\cdot\text{m}^2$), referred to the rear wheels.

In this work, the slip or creep required for a real tire to develop tractive force will be neglected.²⁴ Wheel rotational speed and vehicle velocity will be assumed to be proportional:

$$v_V = r_w \omega_w,$$

where

$$r_w = \text{tire rolling radius (m)}.$$

Using this proportionality in the equation for kinetic energy resolves the vehicle into an equivalent rotating system:

$$E_V = 1/2 I_V \omega_w^2,$$

where

$$I_V = m_V r_w^2 + I_T.$$

External forces can also be resolved into equivalent torques operating on this rotating system:

$$T_{AD} = 1/2 \rho_{\text{air}} C_{DA} r_w^3 \omega_w^2;$$

$$T_{RR} = m_V g r_w C_{RR} \text{ tires};$$

$$T_T = F_T r_w.$$

Writing a continuity equation for energy flow into this rotating system,²⁷ with proper signs for aiding and opposing torques and simplifying:

$$I_V \dot{\omega}_w + D_V \omega_w + 1/2 \rho_{\text{air}} C_{DA} r_w^3 \omega_w^2 + F_T r_w + m_V g r_w C_{RR} \text{ tires} = T_w.$$

The vehicle's rear axle will be assumed ideal with a gear ratio G_{RA} , where

$$\omega_{ps} = G_{RA} \omega_w \text{ and } T_{ps} = T_w / G_{RA}$$

define the torque and speed transformations in the idealized axle between wheel torque and speed (T_w , ω_w) and propeller shaft torque and speed (T_{ps} , ω_{ps}). Using these transformations,

$$\begin{aligned} & \left(\frac{I_V}{G_{RA}^2} \right) \dot{\omega}_{ps} + \left(\frac{D_V}{G_{RA}^2} \right) \omega_{ps} + \left(1/2 \rho_{\text{air}} C_{DA} F \frac{r_w^3}{G_{RA}^3} \right) \omega_{ps}^2 \\ & = T_{ps} - \frac{F_T r_w}{G_{RA}} - \frac{m_V g r_w C_{RR} \text{ tires}}{G_{RA}} \end{aligned}$$

which is a Ricatti equation relating torque in the propeller shaft T_{ps} to its angular velocity ω_{ps} and external torques due to terrain and tire forces.

Obvious extensions of the vehicle model would include:

- 1) adding the effect of nonzero wind velocity,
- 2) coupling of energy into vehicle motion about its pitch axis,²⁶
- 3) better modelling of terrain forces,⁴
- 4) inclusion of tire creep losses, and
- 5) non-ideal behavior of the rear axle.^{23,24}

Inclusion of these effects, however, was felt to be beyond the scope of this work.

It should be noted, parenthetically, that the model assumes a vehicle operating on a roadway. Chassis dynamometers can present loads to the powertrain which differ from those experienced on the road, and a nonquadratic variation of "aero-dynamic" drag with speed.²⁸

Transmission Gearbox Model

The transmission gearbox will be modelled as a single output shaft of inertia I_{To} , connected via ideal clutches to a set of gear reducers, each of which has gear ratio G_{Tk} from input to output and inertia I_{Tk} referred to its input.^{15,25} Gears are assumed ideal. No viscous losses will be assigned to the gearbox at this time, although the method for including them is clear.

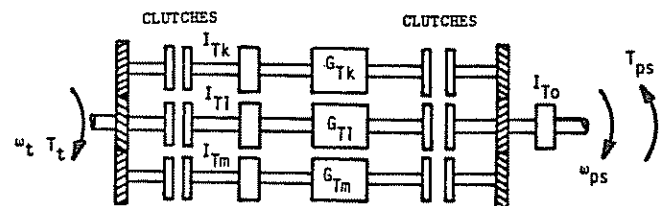


Figure A2 - Transmission Mechanical Model.

With the transmission in the k th gear, the input and output angular velocities are related by the gear ratio G_{Tk} :

$$\omega_t = G_{Tk} \omega_{ps}$$

where ω_t = torque converter turbine angular velocity ($\text{rad}\cdot\text{s}^{-1}$).

The transmission stored kinetic energy is

$$E_{trans} = 1/2 I_{To} \omega_{ps}^2 + 1/2 I_{Tk} \omega_t^2$$

Writing a continuity equation for energy in the gearbox, adding the effect of the vehicle, using the constraint imposed by the gear ratio, and simplifying:

$$\left(\frac{I_{To}}{G_{RA}^2} + I_{Tk} + \frac{I_V}{G_{RA}^2 \cdot G_{Tk}^2} \right) \dot{\omega}_t + \left(\frac{D_V}{G_{RA}^2 \cdot G_{Tk}^2} \right) \omega_t + \left(\frac{1/2 \rho_{air} C_D A_F r_w^3}{G_{RA}^3 \cdot G_{Tk}^3} \right) \omega_t^2 + \frac{F_T r_w}{G_{RA} \cdot G_{Tk}} + \frac{m_v g r_w C_{RR} \text{tires}}{G_{RA} \cdot G_{Tk}} = T_t ;$$

which relates gearbox input speed to the turbine (output) torque of the torque converter.

This is, as before, an inhomogeneous Riccati equation, but with a selectable parameter: the transmission gear ratio G_{Tk} .

The equation for turbine motion will be simplified by the following substitutions, which relate vehicle and gearbox inertia, viscous drag, aerodynamic drag, and external torques to the transmission gear selected.

$$I_k = \frac{I_{To}}{G_{Tk}^2} + I_{Tk} + \frac{I_V}{G_{RA}^2 \cdot G_{Tk}^2}$$

$$D_k = \frac{D_V}{G_{RA}^2 \cdot G_{Tk}^2}$$

$$A_k = \frac{1/2 \rho_{air} C_D A_F r_w^3}{G_{RA}^3 \cdot G_{Tk}^3}$$

$$T_k = \frac{F_T r_w}{G_{RA} \cdot G_{Tk}} + \frac{m_v g r_w C_{RR} \text{Tires}}{G_{RA} \cdot G_{Tk}}$$

In this notation,

$$I_k \dot{\omega}_t + D_k \omega_t + A_k \omega_t^2 + T_k = T_t$$

Engine Model

The model chosen to describe the engine is that of a single rotational inertia I_e , driven by the difference between engine net torque and load torque of the torque converter pump element.

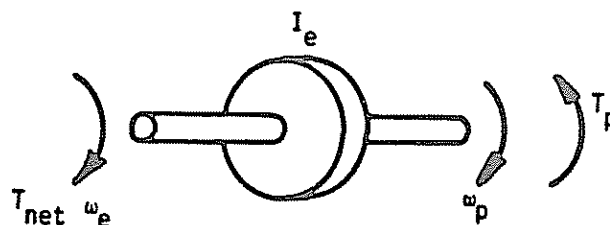


Figure A3 - Engine Model Used for Low Frequency Analysis.

The differential equation describing this system is:

$$I_e \dot{\omega}_e = T_{net} - T_p$$

Indicated, friction, net, and brake torque will be defined at this point to clarify terminology. Indicated torque is the instantaneous torque due to the action of gas forces, without regard to friction.

Friction torque is the torque retarding engine motion in the absence of gas forces.

Net torque is the difference between them - the torque available at any instant to supply power to the load, or to accelerate the engine and load.

Brake torque is net torque measured under conditions of constant time average rotational velocity (no change in average kinetic energy stored in the engine or load).

The assumption made for large signal or low frequency analysis is that specification of all required inputs would produce a well defined net torque, immediately. This is not quite true, because the 4-stroke multi-cylinder engine is a temporally distributed process, combusting mixture taken in one revolution earlier, while pumping against the current manifold pressure. At a 600 r/min idle speed, this can cause 0.2 s transients to occur in our supposed instantaneous process. In the interest of simplicity, these effects will be ignored.

Engine net torque will be approximated as the brake torque produced at steady speed, manifold pressure, and operating condition set by appropriate inputs, operating temperatures, and environmental factors. This allows direct use of steady state engine torque data to drive the model and implicitly accounts for engine viscous damping. Interactions between engine internal flow dynamics and torque production will not be specifically analyzed.^{1,2}

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